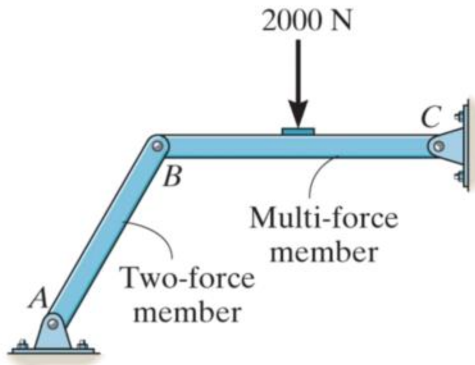
Frames and machines

The general solution method:

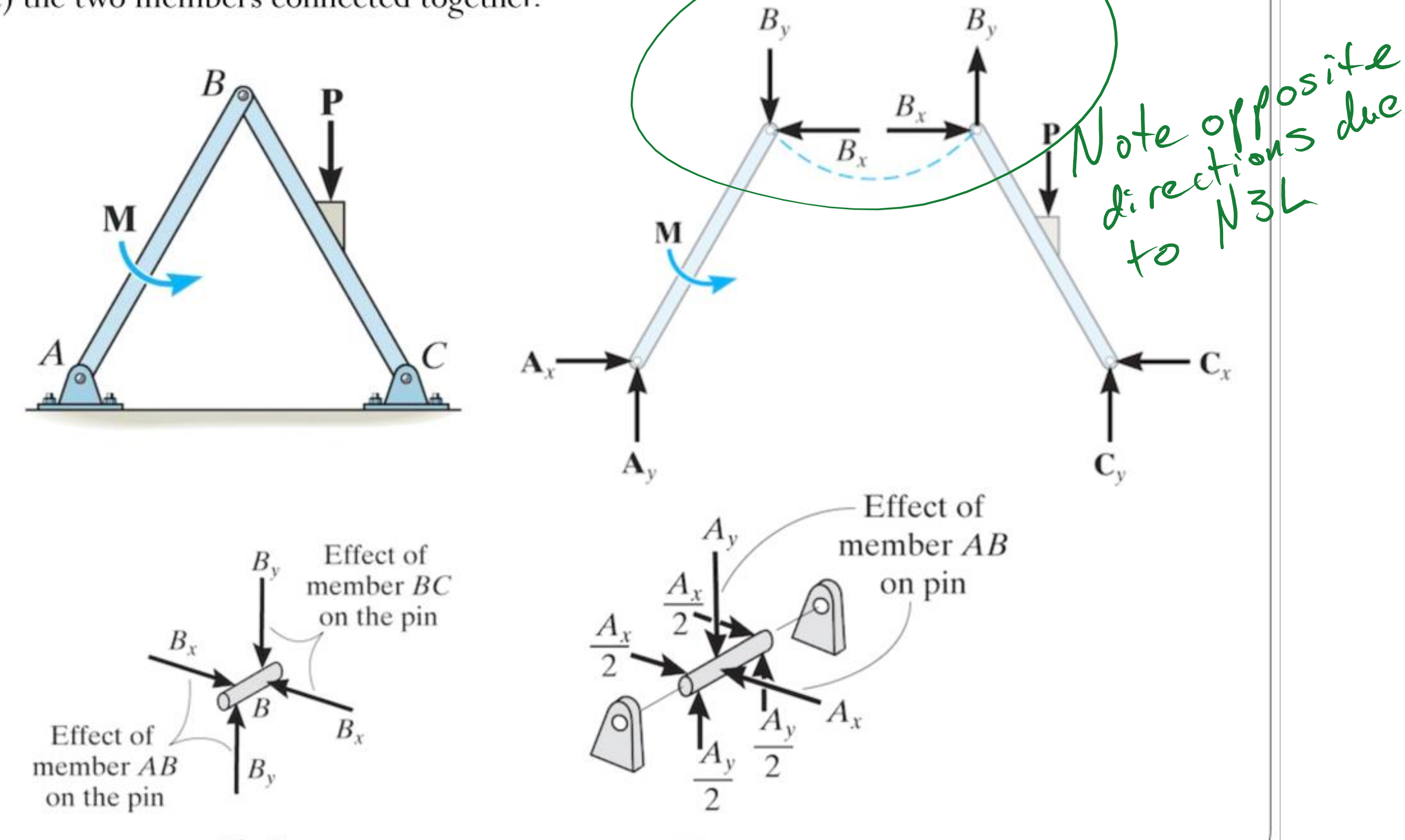
1. Do external equilibrium
   - Find reaction forces



2. Identify two-force members
3. Isolate various parts of the structure (draw their FBD) and analyze equilibrium of them.
   The desired unknowns must appear in at least one FBD!
4. Solve for the requested unknowns.

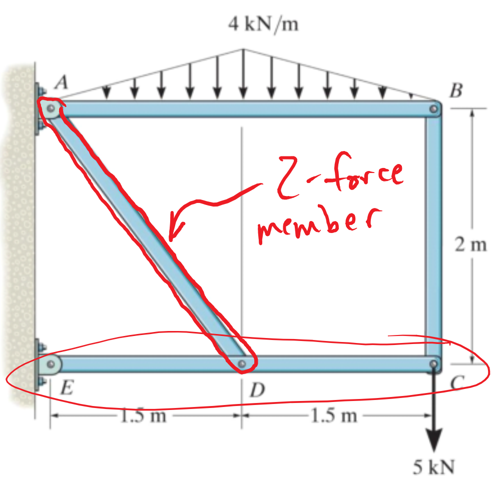
Thursday, February 23, 2017 12:43 PM

For the frame, draw the free-body diagram of (a) each member, (b) the pins at B and A, and (c) the two members connected together.



Pin B

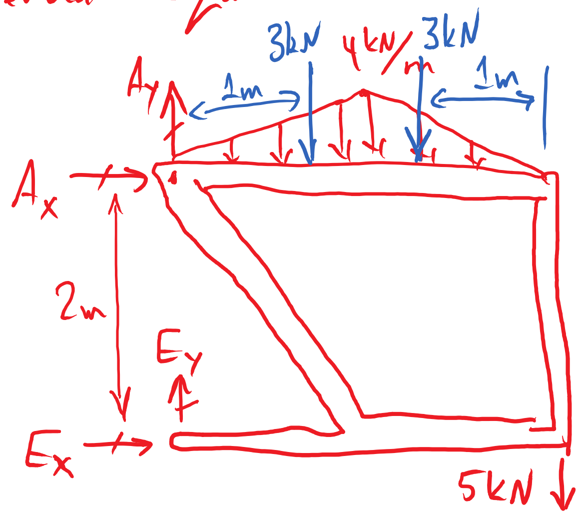
Pin A



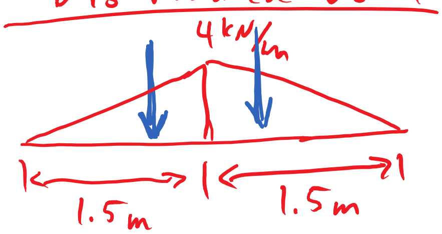
Example 6)

Determine the components of reactions at E and force in member BC

1. External Equilibrium



Distributed Load



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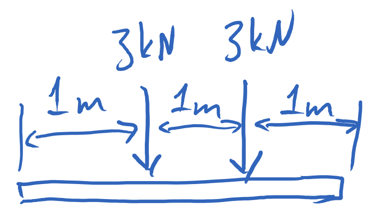


$$(\sum M)_A = 0$$

$$\Rightarrow -(1m)(3kN) - (2m)(3kN) - (3m)(5kN) + (2m)E_x = 0$$

$$\therefore \boxed{E_x = 12\ kN}$$

$$\sum F_x = 0 \Rightarrow A_x + E_x = 0$$

$$\Rightarrow \boxed{A_x = -E_x = -12\ kN}$$

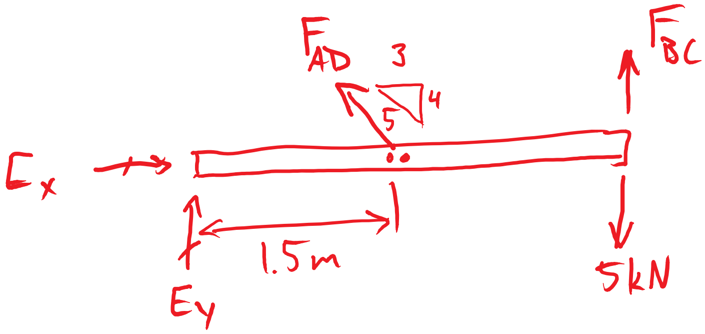
$$\sum F_y = 0 \Rightarrow E_y + A_y - 5kN - 2(3kN) = 0$$

$$E_y + A_y = 11\,kN$$

2. Find any 2-force members

Members AD & BC

3. Equilibrium of EDC



3 unknowns

F_{BC}, F_{AD}, E_y

$$\Sigma F_x = 0 \Rightarrow E_x = F_{AD} \cdot \frac{3}{5} \Rightarrow \boxed{F_{AD} = \frac{5}{3}(E_x) = 20\,kN}$$

$$\Sigma F_y = 0 \Rightarrow E_y + F_{AD} \cdot \frac{4}{5} + F_{BC} - 5\,kN = 0$$

$$E_y + 16\,kN + F_{BC} = 5\,kN$$

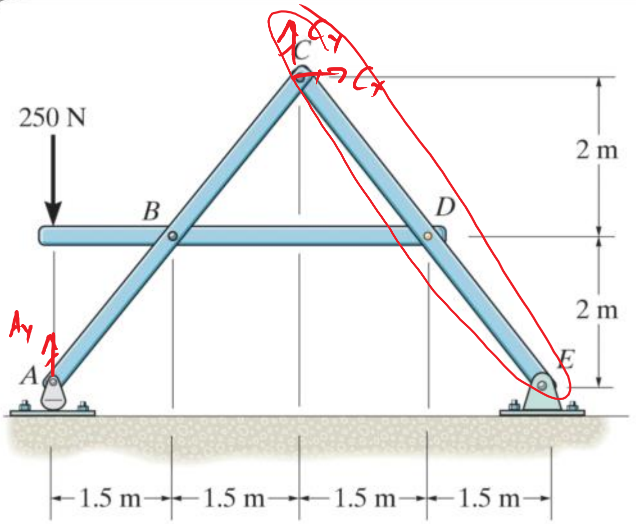
$$E_y + F_{BC} = -11\,kN$$

$$(\Sigma M)_E = 0 \Rightarrow (1.5m)(F_{AD})\frac{4}{5} + (3m)F_{BC} - (3m)(5kN) = 0$$

$$\underbrace{(1.5m)(20kN)\frac{4}{5}}_{24\,kN\cdot m} + (3m)F_{BC} = 15\,kN\cdot m$$

$$24\,kN\cdot m + (3m)F_{BC} = 15\,kN\cdot m$$

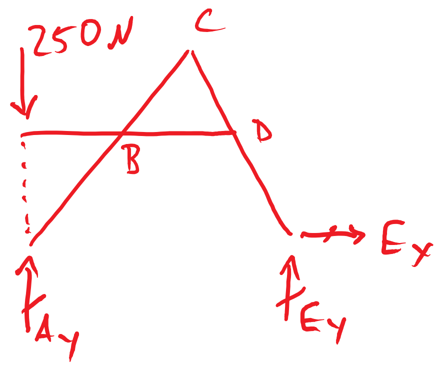
$$\boxed{F_{BC} = -3\,kN}$$



List unknowns

$A_y = 250N$
B_x B_y
C_x C_y
D_x D_y
$E_x = 0$ $E_y = 0$

9



1. External Equilibrium

$(\Sigma M)_E = 0$

$\Rightarrow (6m)(250N) - (6m)A_y = 0$

$\Rightarrow \boxed{A_y = 250N}$

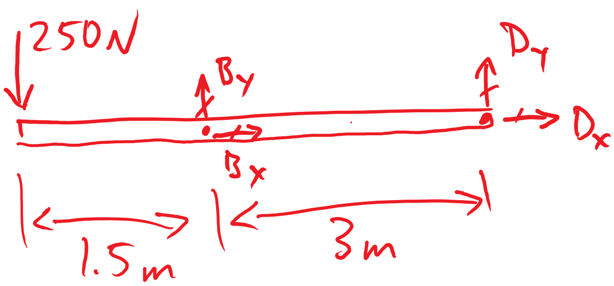
$\Sigma F_y = 0 \Rightarrow -250N + A_y + E_y = 0$

$-250N + 250N + E_y = 0 \Rightarrow \boxed{E_y = 0}$

$\Sigma F_x = \boxed{0 = E_x}$

2. Any 2-force members? No.

3. Equilibrium of BD



$$\left(\sum M\right)_B = 0$$

$$\Rightarrow (1.5m)(250N) + (3m)\cdot D_y = 0$$

$$\Rightarrow \boxed{D_y = -125N}$$

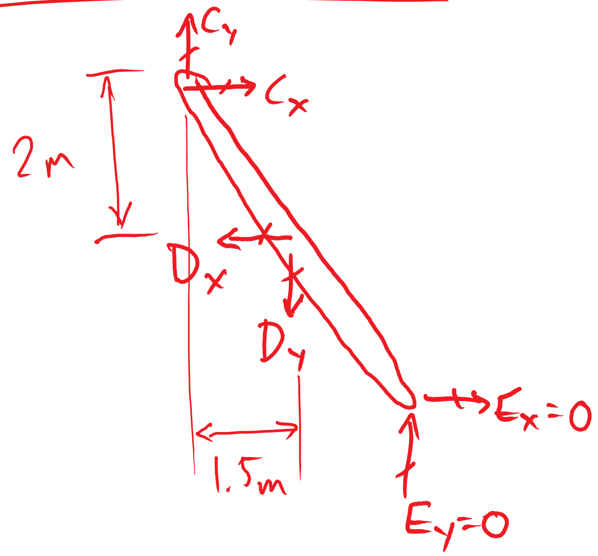
$$\sum F_y = 0 \Rightarrow -250N + B_y + D_y = 0$$

$$(-250N) + B_y + (-125N) = 0$$

$$\Rightarrow \boxed{B_y = 375N}$$

$$\sum F_x = 0 \Rightarrow B_x = -D_x$$

Equilibrium of CDE



$$\left(\sum M\right)_C = 0$$

$$-(2m)D_x - (1.5m)D_y = 0$$

$$D_x = -D_y\frac{1.5}{2} = -(-125N)\frac{1.5}{2}$$

$$\boxed{D_x = 93.75N}$$

$$\sum F_x = 0$$

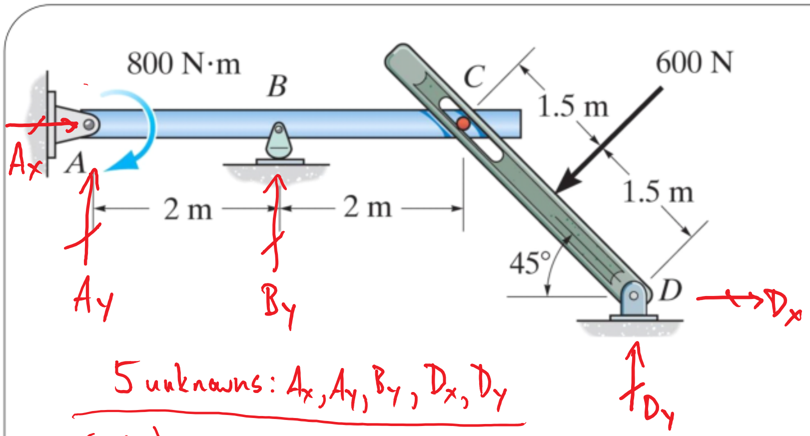
$$\Rightarrow C_x - D_x = 0 \Rightarrow \boxed{C_x = 93.75N}$$

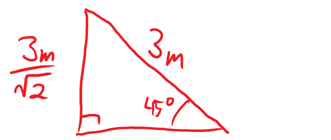
$\sum F_x = 0$

$\Rightarrow \quad C_x - D_x = 0 \Rightarrow \boxed{C_x = 93.75N}$

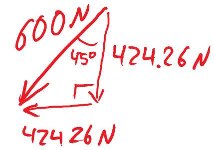
$\sum F_y = 0 \Rightarrow C_y - D_y = 0$

$\boxed{C_y = D_y = -125N}$





$\frac{3m}{\sqrt{2}} \approx 2.1213m$



5 unknowns: A_x, A_y, B_y, D_x, D_y

$(\Sigma M)_D = 0$

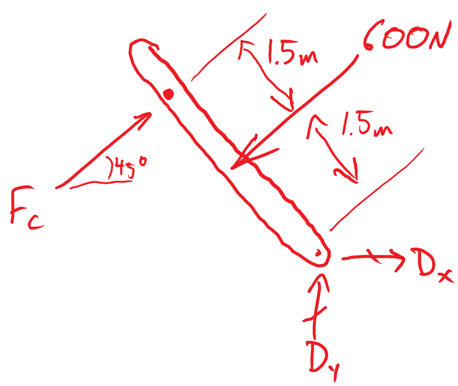
$\Rightarrow -800N.m + (1.5m)(600N) - (4.1213m)B_y - (6.1214m)\cdot A_y - (2.1213m)\cdot A_x = 0$

$\Rightarrow (4.1213m)B_y + (6.1213m)A_y + (2.1213m)A_x = 100N\cdot m$

$\Sigma F_x = 0 \Rightarrow A_x + D_x - 424.26N = 0$

$\Sigma F_y = 0 \Rightarrow A_y + B_y + D_y - 424.26N = 0$

FBD of CD



$(\Sigma M)_c = 0$

$\Rightarrow -(1.5m)(600N) + (2.1213m)D_y + (2.1213m)D_x = 0$

$\Rightarrow D_x + D_y = \frac{900N\cdot m}{2.1213m} = 424.26N$

$\Sigma F_x = 0$

$\Rightarrow F_c \cdot \frac{\sqrt{2}}{2} + D_x = 424.26N$ ☆

$\Sigma F_y = 0$

$\Rightarrow F_c \frac{\sqrt{2}}{2} + D_y = 424.26N$ ☆☆

solve these 3 eqns. in 3 unknowns (F_c, D_x, D_y)

Subtract ☆☆ from ☆ to get:

$D_x - D_y = 0 \Rightarrow D_x = D_y$

Then, $(\Sigma M)_c$ gives $D_x + D_y = 424.26\,N$

$$\Rightarrow 2D_x = 424.26\,N$$

$$\boxed{D_x = 212\,N}$$

$$\boxed{D_y = 212\,N}$$

$$\Rightarrow \boxed{F_c = \sqrt{2}\,(424.26\,N - 212.13\,N) = 300\,N}$$

From external equilibrium (step 1):

$$\Sigma F_x = 0 \Rightarrow A_x + D_x = 424.26\,N \Rightarrow \boxed{A_x = 212\,N}$$

$$\Sigma F_y = 0 \Rightarrow A_y + B_y + D_y = 424.26\,N$$

$$A_y + B_y = 424.26\,N - D_y = 424.26 - 212.13\,N$$

$$\Rightarrow A_y + B_y = 212.13\,N \quad \text{☺}$$

$$(\Sigma M)_D = 0$$

$$\Rightarrow (4.1213\,m)B_y + (6.1213\,m)A_y + (2.1213\,m)A_x = 100\,N\cdot m$$

$$(4.1213\,m)B_y + (6.1213\,m)A_y + (2.1213\,m)(212.13\,N) = 100\,N\cdot m$$

$$(4.1213\,m)B_y + (6.1213\,m)A_y = 349.99\,N m \quad \text{☹}$$

Solve the system ☺ & ☹ to get:

$$(4.1213\,m)(212.13\,N - A_y) + (6.1213\,m)A_y = 349.99\,N\cdot m$$

$$874.25\,N\cdot m + (2\,m)\cdot A_y = 349.99\,N\cdot m$$

$$\Rightarrow A_y = \frac{-524.26\,N\cdot m}{2\,m} = 262.13\,N$$

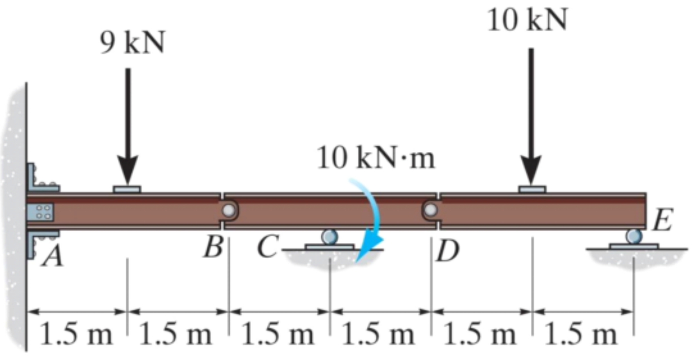
$$\boxed{A_y = 262\,N}$$

$B_y = 212.13N - A_y = 212.13N - (262.13N)$

$$\boxed{B_y = -50\,N}$$

Page 33

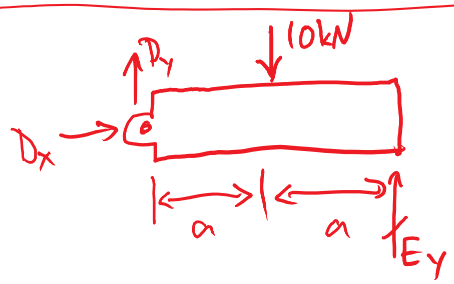
Thursday, February 23, 2017 12:43 PM



Let $a = 1.5\,m$

No two-force members

Start with member DE



$$(\Sigma M)_D = 0$$

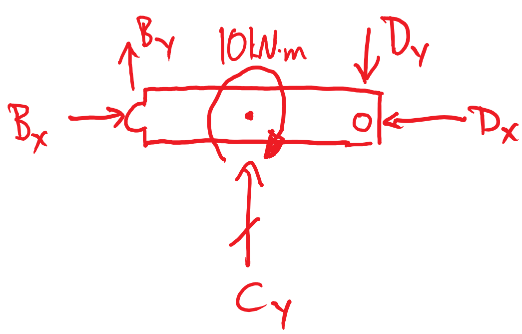
$$\Rightarrow -a \cdot (10\,kN) + 2a E_y = 0$$

$$\Rightarrow \boxed{E_y = 5\,kN}$$

$$\Sigma F_x = 0 \Rightarrow \boxed{D_x = 0}$$

$$\Sigma F_y = 0 \Rightarrow D_y - 10\,kN + 5\,kN = 0 \Rightarrow \boxed{D_y = 5\,kN}$$

Next, member BCD



$$(\Sigma M)_C = 0$$

$$-a \cdot B_y - a \cdot D_y - 10\,kN \cdot m = 0$$

$$\Rightarrow B_y + D_y = \frac{-10\,kN \cdot m}{1.5\,m} = -6.667\,kN$$

$$B_y + (5\,kN) = -6.667\,kN$$

$$\Rightarrow \boxed{B_y = -11.7\,kN}$$

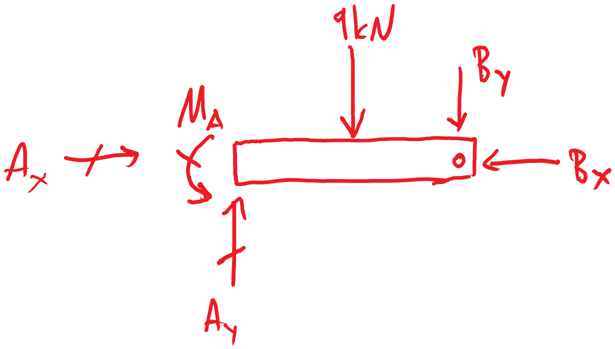
$$\Sigma F_x = 0 \Rightarrow B_x - D_x = 0 \Rightarrow \boxed{B_x = D_x = 0}$$

$$\Sigma F_y = 0 \Rightarrow B_y - D_y + C_y = 0$$

$$\sum F_y = 0 \Rightarrow B_y - D_y + C_y = 0$$

$$(-11.667\,kN) - (5\,kN) + C_y = 0 \Rightarrow \boxed{C_y = 16.7\,kN}$$

Next, member AB



$$(\sum M)_A = 0$$

$$\Rightarrow M_A - a(9kN) - 2a \cdot B_y = 0$$

$$M_A = (1.5m)(9kN) + 2(1.5m)B_y$$

$$M_A = 13.5\,kN \cdot m + (3m)(-11.667\,kN)$$

$$\boxed{M_A = -21.5\,kN \cdot m}$$

$$\sum F_x = 0 \Rightarrow 0 = A_x - B_x \Rightarrow \boxed{A_x = B_x = 0}$$

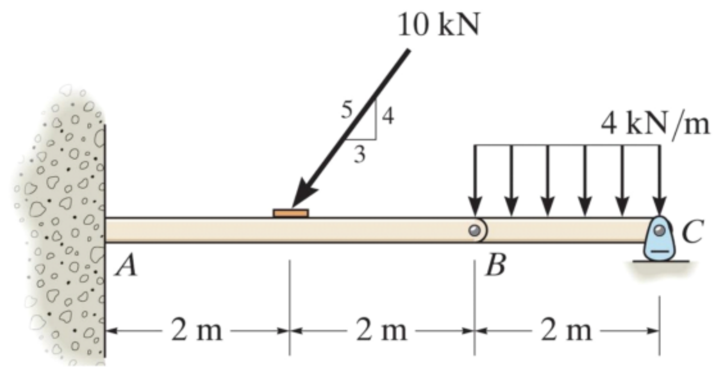
$$\sum F_y = 0 \Rightarrow A_y - 9kN - B_y = 0$$

$$\Rightarrow A_y = 9kN + B_y = 9kN + (-11.667\,kN)$$

$$\boxed{A_y = -2.67\,kN}$$

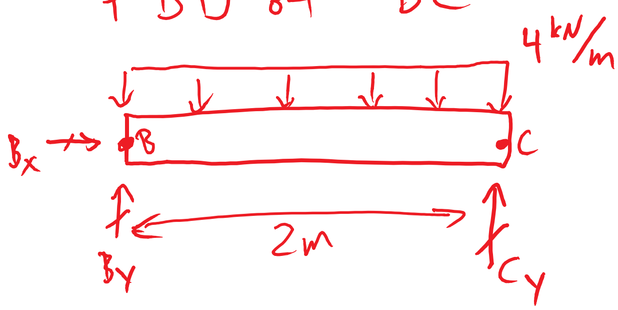
Example 7)

The compound beam shown is pin-connected at B. Determine the components of reaction at its supports. Neglect its weight and thickness.



No 2-force members

FBD of BC:



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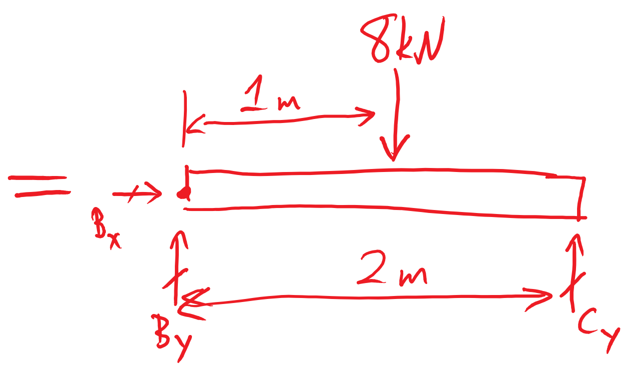


$(\Sigma M)_B = 0$

$\Rightarrow -(1m)(8kN) + (2m)\cdot C_y = 0$

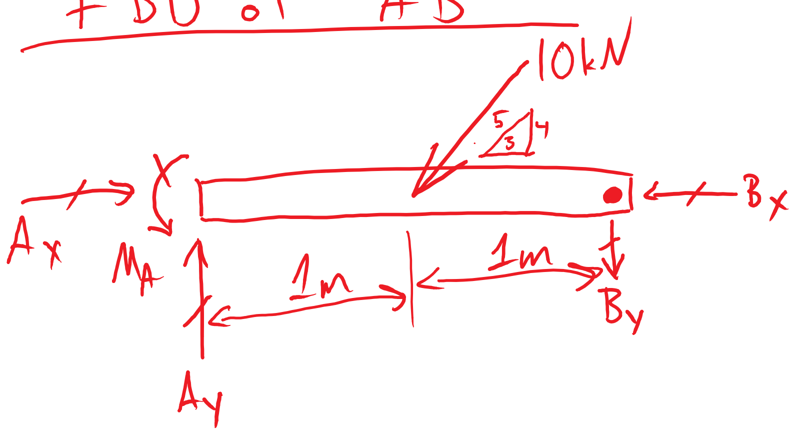
$\Rightarrow \boxed{C_y = 4kN}$

$\Sigma F_x = 0 \Rightarrow \boxed{B_x = 0}$

$\Sigma F_y = 0 \Rightarrow B_y + C_y = 8kN$

$\Rightarrow \boxed{B_y = 4kN}$

FBD of AB



$\Sigma F_x = 0$

$\Rightarrow A_x - (10kN)\frac{3}{5} - \cancel{B_x}^{\,0} = 0$

$\Rightarrow \boxed{A_x = 6\ kN}$

$$\Sigma F_y = 0$$

$$\Rightarrow A_y - (10\,kN)\tfrac{4}{5} - B_y = 0$$

$$A_y = (8\,kN) + B_y$$

$$= 8\,kN + 4\,kN$$

$$\Rightarrow \boxed{A_y = 12\,kN}$$

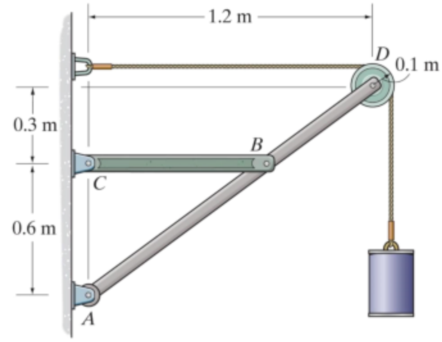
$$(\Sigma M)_A = 0$$

$$\Rightarrow M_A - (1\,m)(10\,kN)\tfrac{4}{5} - (2\,m)B_y = 0$$

$$M_A = 8\,kN\cdot m + (2\,m)(4\,kN)$$

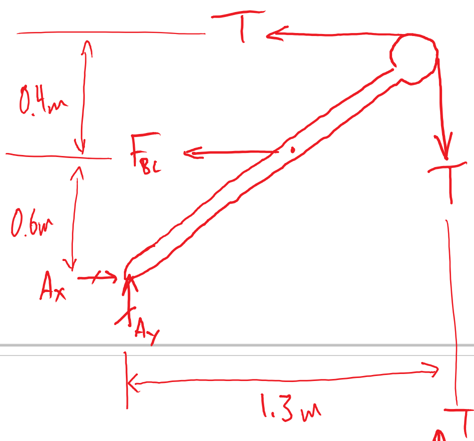
$$\boxed{M_A = 16\,kN\cdot m}$$

Example 8)
The frame supports a 50kg cylinder. Determine the horizontal and vertical components of reaction at A and the force at C



Note that BC is a two-force member

FBD of ABD



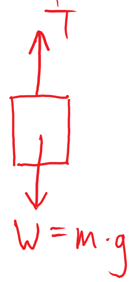
$$\left(\sum M\right)_A = 0$$

$$\Rightarrow (0.6m)\cdot F_{BC} + (1.0m)\cdot T - (1.3m)\cdot T = 0$$

$$\Rightarrow (0.6)F_{BC} - (0.3)\cdot T = 0$$

$$\Rightarrow F_{BC} = \frac{1}{2}T$$

FBD of weight:



$$\Rightarrow T = m\cdot g = (50\,kg)(9.81\,\tfrac{m}{s^2})$$

$$T = 490.5\,N$$

$$\boxed{T = 491\,N}$$

$$\Rightarrow F_{BC} = \frac{490.5\,N}{2} = 245.25\,N$$

$$\boxed{F_{BC} = 245\,N}$$

$$\sum F_x = 0 \Rightarrow A_x - F_{BC} - T = 0$$

$$\Rightarrow A_x = F_{BC} + T$$

$$A_x = (245.25\,N) + (490.5\,N)$$

$$A_x = 735.75\,N$$

$$\boxed{A_x = 736\,N}$$

$\sum F_y = 0 \Rightarrow A_y - T = 0 \quad \Rightarrow \boxed{A_y = 491\,N}$

A 500 kg elevator car is being hoisted by a motor using a pulley system. If the car travels at a constant speed, determine the force developed in the cables. Neglect the cable and pulley masses.

We'll label the tension in the rightmost cable T_1, and tension in the leftmost cable T_2. Which is an equation for equilibrium of pulley C?

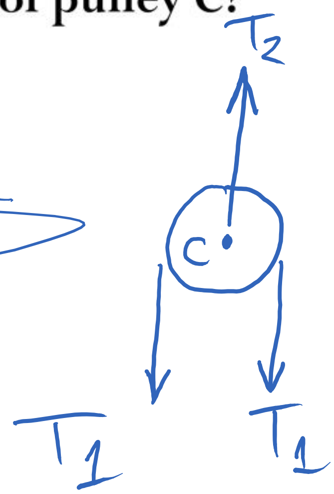
A. $T_1+2T_2 = 0$

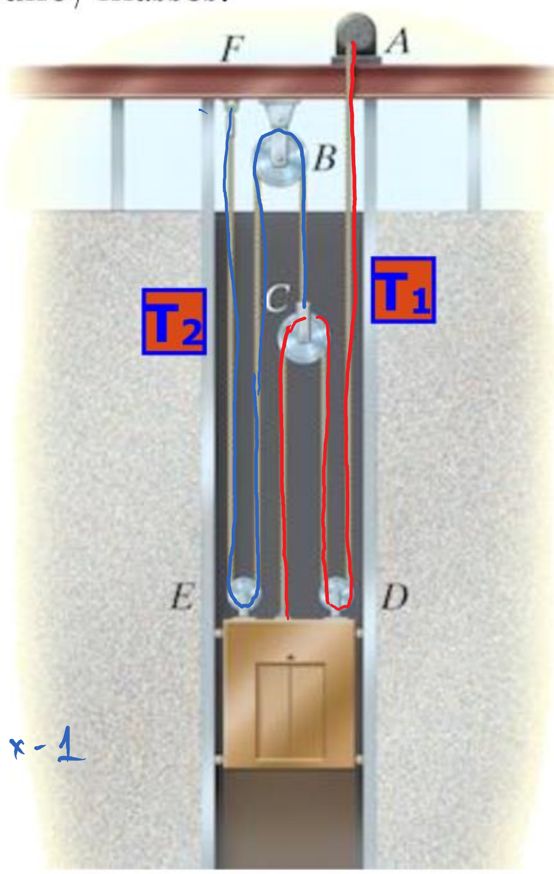
B. $2T_1+T_2 = 0$

C. $T_1-T_2 = 0$

D. $2T_1-T_2 = 0$

E. $T_1-2T_2 = 0$





$$\Sigma F_y = 0 = (T_2 - 2T_1) \times -1$$

$$2T_1 - T_2 = 0$$

A 500 kg elevator car is being hoisted by a motor using a pulley system. If the car travels at a constant speed, determine the force developed in the cables. Neglect the cable and pulley masses.

We'll label the tension in the rightmost cable T_1, and tension in the leftmost cable T_2. Which is an equation for equilibrium of the car?

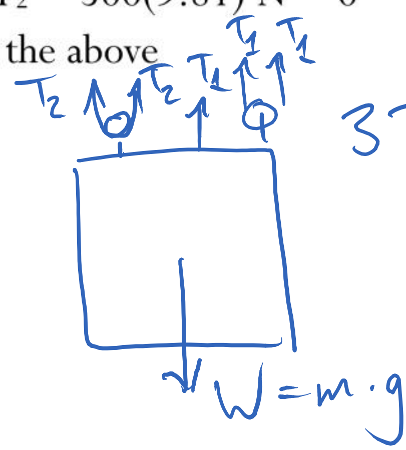
A. $3T_1 + 2T_2 + 500(9.81)\text{ N} = 0$

B. $3T_1 - 4T_2 + 500(9.81)\text{ N} = 0$

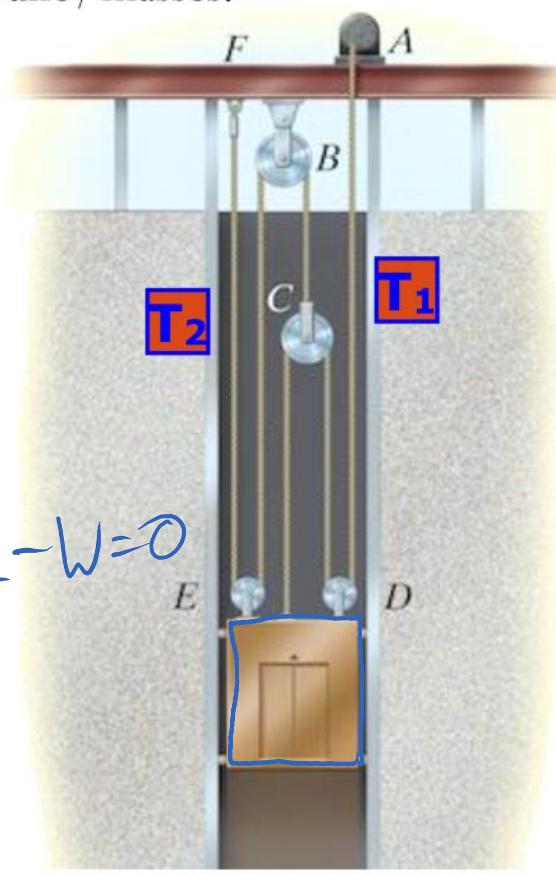
C. $3T_1 + 2T_2 - 500(9.81)\text{ N} = 0$

D. $3T_1 - 2T_2 - 500(9.81)\text{ N} = 0$

E. None of the above

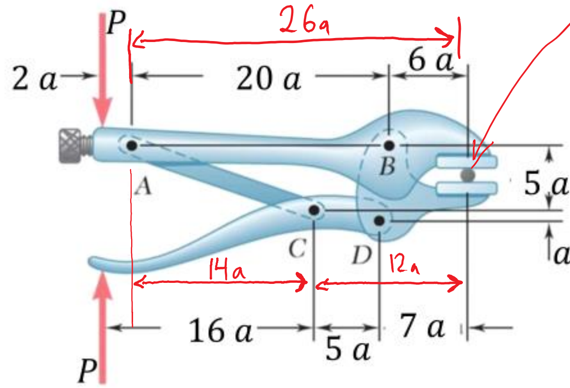


$3T_1 + 2T_2 - W = 0$

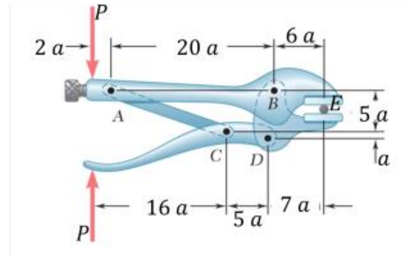


Thursday, February 23, 2017 12:43 PM

goal: find force here



Determine the magnitude of the gripping forces produced when the force P is applied as shown.



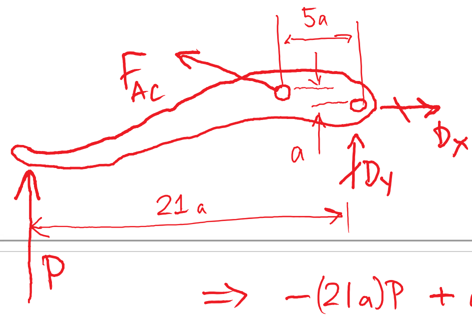
See that AC is a two-force member

Find geometry of AC.

$26a - 12a = 14a$

$\sqrt{5^2 + 14^2} = \sqrt{221}$

FBD of lower handle



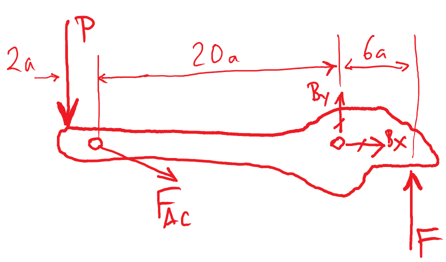
$(\Sigma M)_D = 0$

$$\Rightarrow -(21a)P + a \cdot F_{AC} \cdot \frac{14}{\sqrt{221}} - (5a) F_{AC} \cdot \frac{5}{\sqrt{221}} = 0$$

$$-21\cancel{a} \cdot P + \cancel{a} \cdot F_{AC} \cdot \frac{14-25}{\sqrt{221}} = 0$$

$$\Rightarrow \boxed{F_{AC} = \frac{-\sqrt{221}}{11} \cdot 21 \cdot P}$$

Now that F_{AC} is known, the FBD of the upper handle should lead to the clamping force.



$(\Sigma M)_B = 0$

$$(22a) \cdot P + (6a) \cdot F + (20a) F_{AC} \cdot \frac{5}{\sqrt{221}} = 0$$

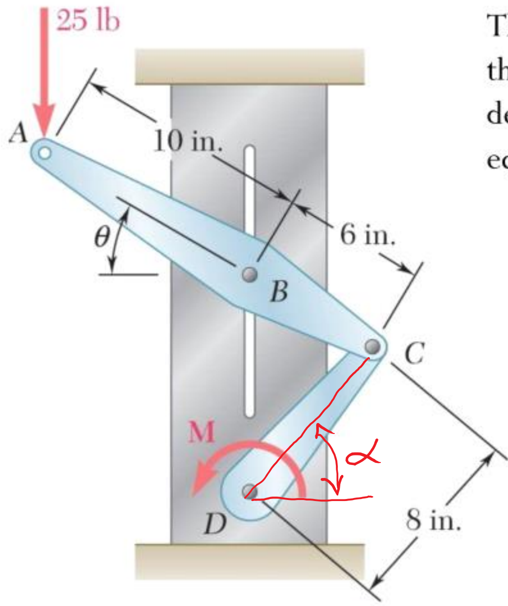
$$6F = -22P - \frac{100}{\sqrt{221}} \cdot F_{AC}$$

$$F = -\frac{22}{6} P - \frac{50}{3\cancel{\sqrt{221}}} \left(-\frac{21}{11} \cancel{\sqrt{221}} \cdot P \right)$$

$$F = P \cdot \left(-\frac{11}{3} + \frac{350}{11} \right)$$

$$F = P \cdot \left(\frac{-121 + 1050}{33} \right)$$

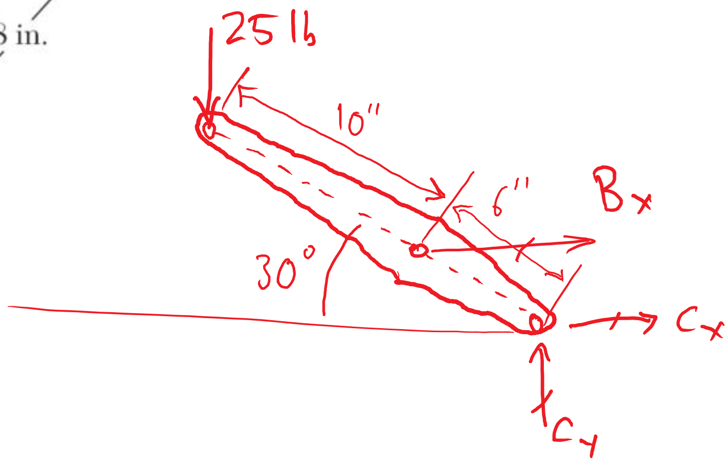
$$\boxed{\begin{aligned} F &= \frac{929}{33} \cdot P \\ F &\approx 28.2 \cdot P \end{aligned}}$$



The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple $\mathbf{M}$ required to hold the system in equilibrium when $\theta = 30°$.

Note: No two-force members.

FBD of ABC



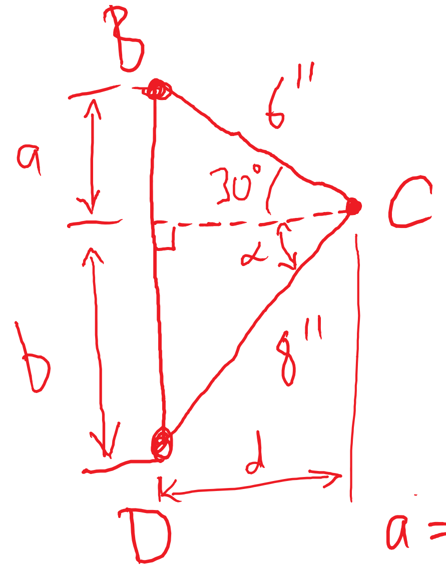
$$(\Sigma M)_C = 0 = (16'' \cos 30°)(25\text{ lb}) - (6'' \cdot \sin 30°) B_x = 0$$

$$B_x = \frac{16 \cos(30°)}{6 \cdot \sin(30°)} \cdot (25\text{ lb}) = \frac{8}{3} \cdot \sqrt{3} \cdot (25\text{ lb})$$

$$B_x = 115.5\text{ lb} \qquad \boxed{B_x = 116\text{ lb}}$$

$$\Sigma F_x = 0 \Rightarrow B_x + C_x = 0 \Rightarrow \boxed{C_x = -B_x = -116\text{ lb}}$$

$$\Sigma F_y = 0 \Rightarrow C_y - 25\text{ lb} = 0 \Rightarrow \boxed{C_y = 25\text{ lb}}$$



Find distance (a+b) from D to B

$$a = (6'') \cdot \sin(30°) = \tfrac{1}{2} \cdot (6'') = 3''$$

$$d = 6'' \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}'' = 5.196''$$

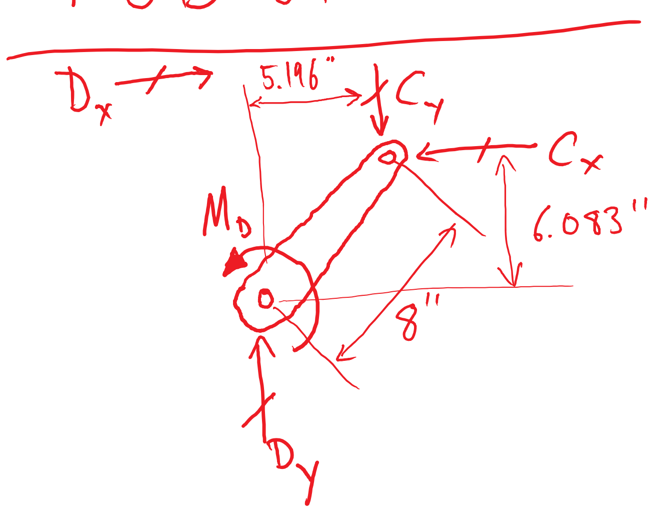
Find α. $\cos(\alpha) = d/8'' = 5.196''/8''$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{5.196}{8}\right) = 49.5°$$

$$\Rightarrow b = 8'' \cdot \sin(\alpha) = 8'' \cdot \sin(49.5°) = 6.083''$$

$$(a+b) = 9.083''$$

FBD of CD



$$\left(\sum M\right)_D = 0 = M_D - (5.196'')C_y + (6.083'')\cdot C_x = 0$$

$$\Rightarrow M_D = (5.196'')(25\text{ lb}) - (6.083'')(-116\text{ lb})$$

$$M_D = 129.9\text{ lb-in} + 705.6\text{ lb-in}$$

$$M_D = 835.5\text{ lb-in}$$

$$\boxed{M_D = 836\text{ lb-in}}$$